Unit-1

Rayleigh and Ricean Distribution

- Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelop of an individual multipath component.

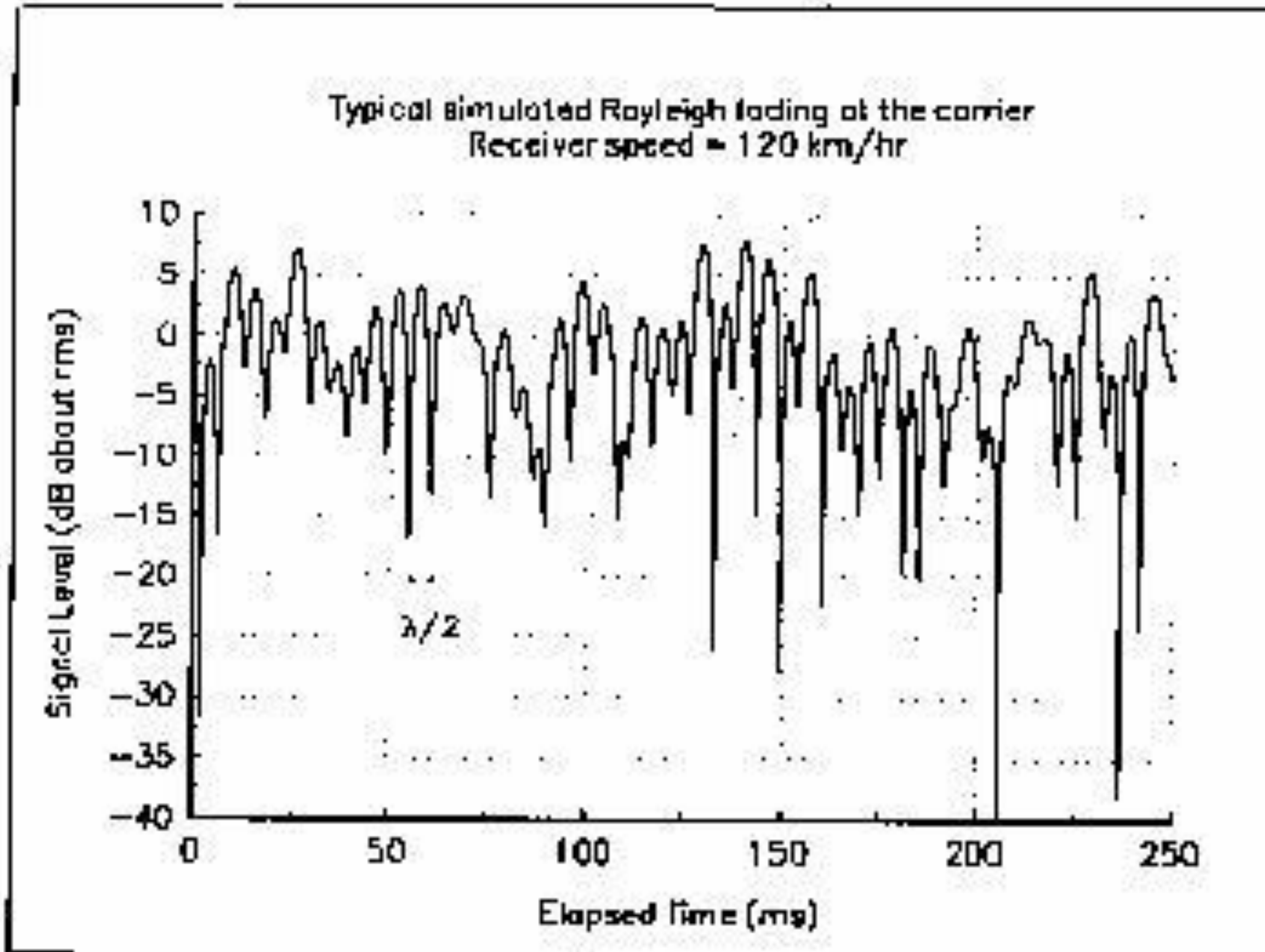

Figure 5.15
A typical Rayleigh fading envelope at 900 MHz [From [Fun93] © IEEE].

- The Rayleigh distribution has a pdf given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases} \quad (5.49)$$

whe σ

σ^2 is the rms value of the received signal

is the time-average power of the received signal.

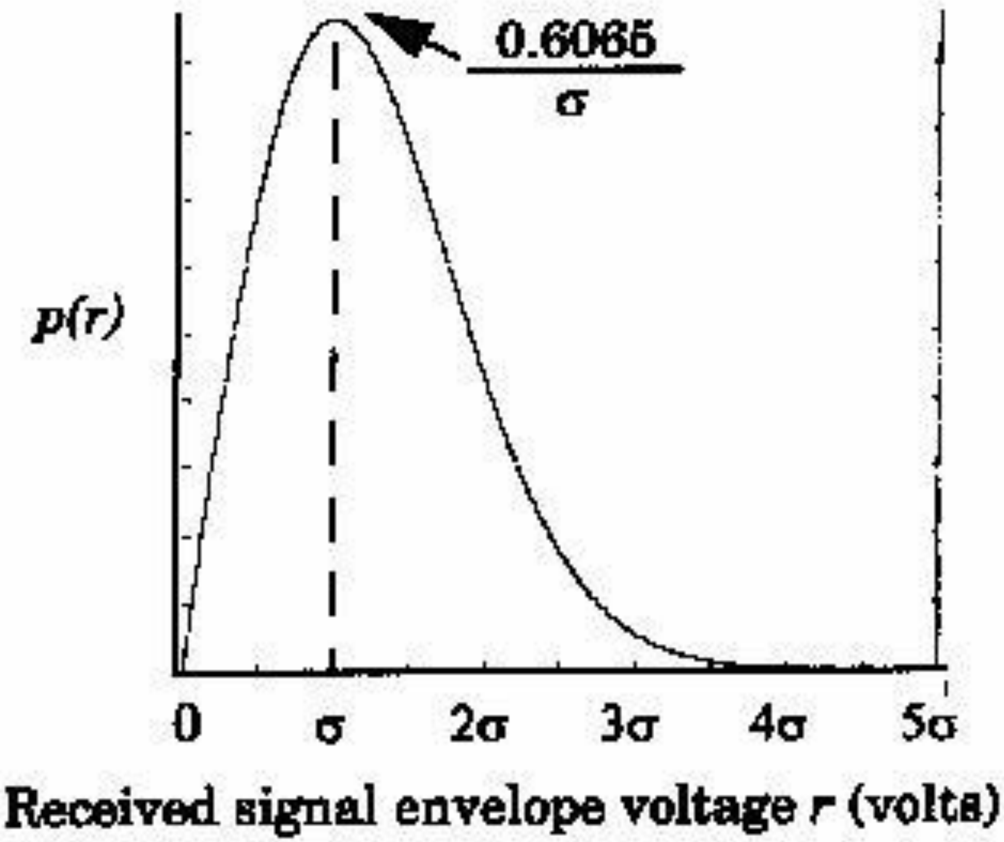

Figure 5.16
Rayleigh probability density function (pdf).

- When there is a dominant stationary (non-fading) signal component present, such as a Line-of-sight propagation path, the small-scale fading envelope distribution is Ricean.

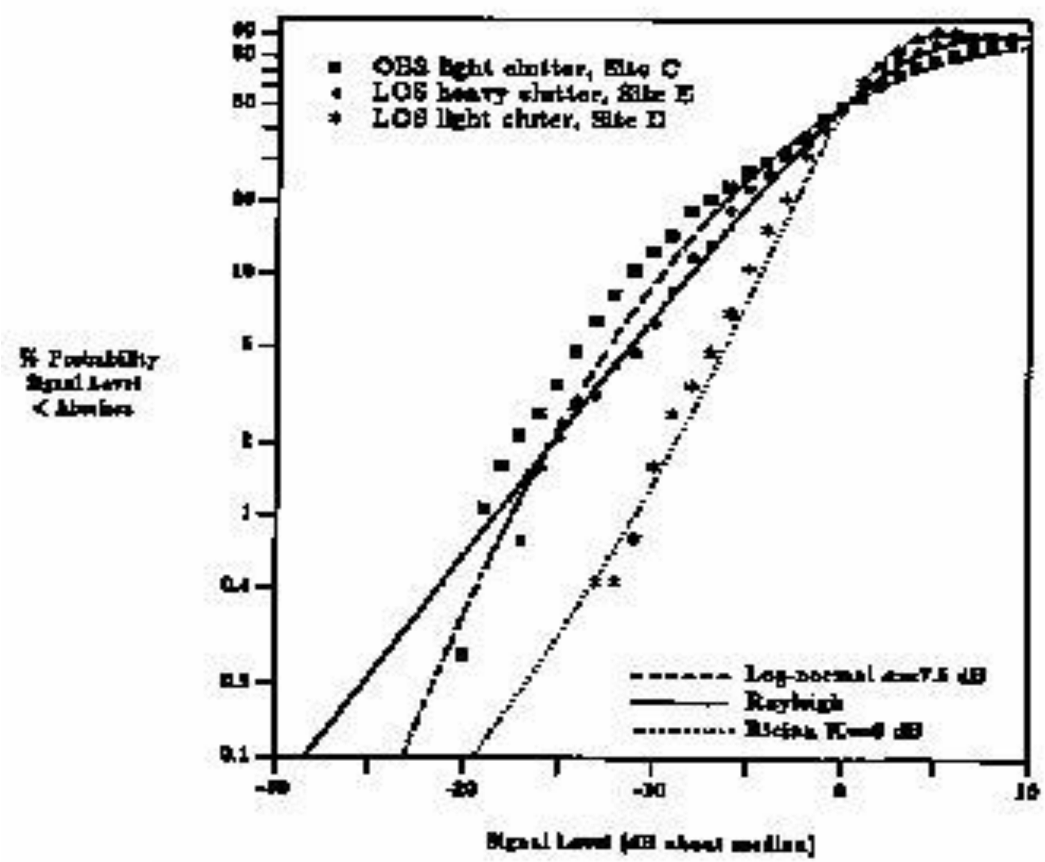

Figure 5.17
Cumulative distribution for three small-scale fading measurements and their fit to Rayleigh, Ricean, and log-normal distributions [From [Rap89] © IEEE].

5.7 Statistical Model for Multipath Fading channels

- Like that of the large-scale modeling, several multipath models have been suggested to explain the observed statistical nature of a mobile channel.

- Clarke's fading model

- Level crossing rate (LCR) and average fade duration of a Raleigh fading signal.

Example 5.7

For a Rayleigh fading signal, compute the positive-going level crossing rate for $\rho = 1$, when the maximum Doppler frequency (f_m) is 20 Hz. What is the maximum velocity of the mobile for this Doppler frequency if the carrier frequency is 900 MHz?

Solution to Example 5.7

Using equation (5.80), the number of zero level crossings is

$$N_R = \sqrt{2\pi}(20)(1)e^{-1} = 18.44 \text{ crossings per second}$$

The maximum velocity of the mobile can be obtained using the Doppler relation, $f_{d,max} = v/\lambda$.

Therefore velocity of the mobile at $f_m = 20$ Hz is

$$v = f_d\lambda = 20 \text{ Hz}(1/3 \text{ m}) = 6.66 \text{ m/s} = 24 \text{ km/hr}$$

Example 5.8

Find the average fade duration for threshold levels $\rho = 0.01$, $\rho = 0.1$, and $\rho = 1$, when the Doppler frequency is 200 Hz.

Solution to Example 5.8

Average fade duration can be found by substituting the given values in equation 5.75

For $\rho = 0.01$

$$\bar{\tau} = \frac{e^{0.01^2} - 1}{(0.01)\,200\sqrt{2\pi}} = 19.9\mu s$$

For $\rho = 0.1$

$$\bar{\tau} = \frac{e^{0.1^2} - 1}{(0.1)\,200\sqrt{2\pi}} = 200\mu s$$

For $\rho = 1$

$$\bar{\tau} = \frac{e^{1^2} - 1}{(1)\,200\sqrt{2\pi}} = 3.43 \text{ ms}$$

Example 5.9

Find the average fade duration for a threshold level of $\rho = 0.707$ when the Doppler frequency is 20 Hz. For a binary digital modulation with bit duration of 50 bps, is the Rayleigh fading slow or fast? What is the average number of bit errors per second for the given data rate. Assume that a bit error occurs whenever any portion of a bit encounters a fade for which $\rho < 0.1$.

Solution to Example 5.9

The average fade duration can be obtained using equation (5.84).

$$\bar{\tau} = \frac{e^{0.707^2} - 1}{(0.707)\, 20\sqrt{2\pi}} = 18.3 \text{ ms}$$

For a data rate of 50 bps, the bit period is 20 ms. Since the bit period is greater than the average fade duration, for the given data rate the signal undergoes fast Rayleigh fading. Using equation (5.84), the average fade duration for $\rho = 0.1$ is equal to 0.002 s. This is less than the duration of one bit. Therefore, only one bit on average will be lost during a fade. Using equation (5.80), the number of level crossings for $\rho = 0.1$ is $N_r = 4.96$ crossings per seconds. Since a bit error is assumed to occur whenever a portion of a bit encounters a fade, and since average fade duration spans only a fraction of a bit duration, the total number of bits in error is 5 per second, resulting in a BER = (5/50) = 0.1.

- Two-ray Raleigh Fading Model

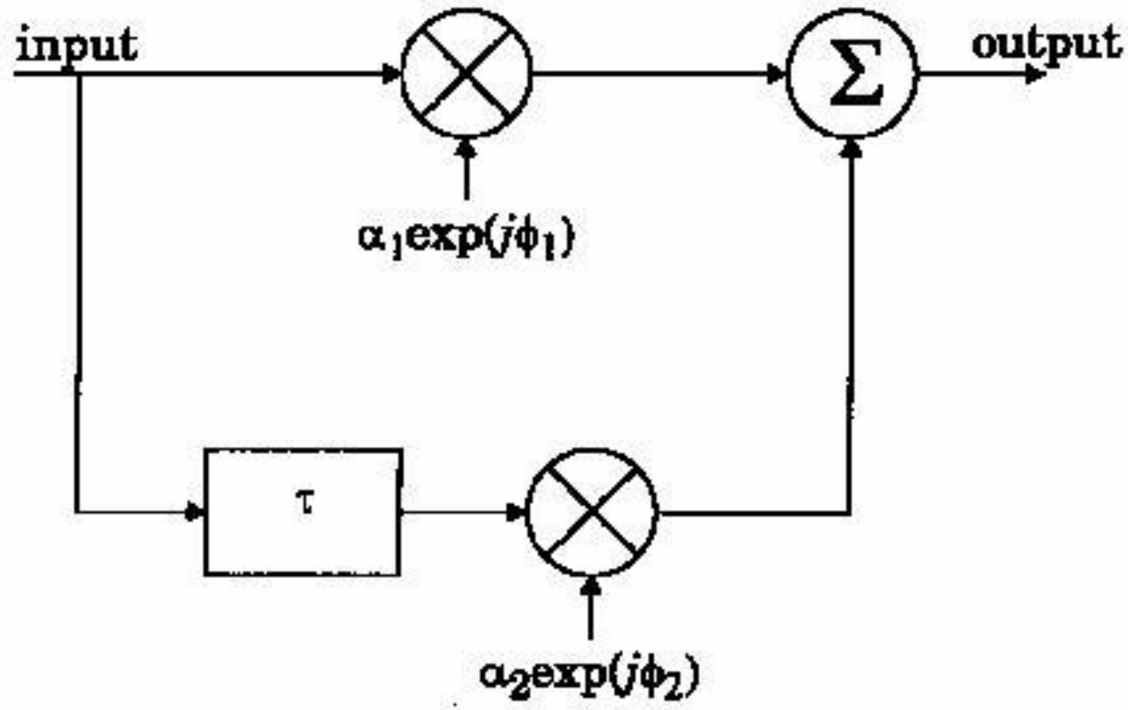

Figure 5.26
Two-ray Rayleigh fading model